Secondary Two Mathematics: An Integrated Approach
Module 9
Probability

By

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Module 9 – Probability

Classroom Task: 9.0 Tree Diagrams and Two way Frequency Tables – Review materials.
Two activities for in class and Two for Homework

Classroom Task: 9.1 TB or Not TB – A Develop Understanding Task
*Estimating conditional probabilities and interpreting the meaning of a set of data (S.CP.6, S.MD.7+)*
Ready, Set, Go Homework: Probability 9.1

Classroom Task: 9.2 Chocolate versus Vanilla – A Solidify Understanding Task
*Examining conditional probability using multiple representations (S.CP.6)*
Ready, Set, Go Homework: Probability 9.2

Classroom Task: 9.3 Fried Freddy’s – A Solidify Understanding Task
*Using sample to estimate probabilities (S.CP.2, S.CP.6)*
Ready, Set, Go Homework: Probability 9.3

Classroom Task: 9.4 Visualizing with Venn – A Solidify Understanding Task
*Creating Venn diagram’s using data while examining the addition rule for probability (S.CP.6, S.CP.7)*
Ready, Set, Go Homework: Probability 9.4

Classroom Task: 9.5 Freddy Revisited – A Solidify Understanding Task
*Examining independence of events using two-way tables (S.CP.2, S.CP.3, S.CP.4, S.CP.5)*
Ready, Set, Go Homework: Probability 9.5

Classroom Task: 9.6 Striving for Independence – A Practice Understanding Task
*Using data in various representations to determine independence (S.CP.2, S.CP.3, S.CP.4, S.CP.5)*
Ready, Set, Go Homework: Probability 9.6

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Topic: Tree Diagrams- Worksheet 1

1. A dice numbered 1 to 4 is rolled and 1 coin is tossed. Draw a tree diagram to represent the possible outcomes.

2. Draw a tree diagram to represent total outcomes for flipping 2 Pennies.

3. Three colors of cars that are In red, blue and white color is driven simultaneously. Draw a tree diagram to represent the possible outcomes.

4. A bag contains yellow balls numbered 1 to 4, blue balls numbered 1 to 3, and pink balls numbered 1 to 2. Draw a tree diagram to represent total outcomes for selecting two balls.

5. Draw a tree diagram to represent total outcomes for flipping a Nickel.

6. 1 three-sided numbered cube is rolled. Draw a tree diagram to represent total outcomes?

7. 1 quartile and one five sides dice numbered from 1 to 5 are tossed. Draw as tree diagram to illustrate total outcomes.

8. If a shopkeeper has 2 kinds of pencils in 2 different brands. Draw a tree diagram to select.

9. A man has 2 places to visit. Draw a tree diagram to represent the possible outcomes.

10. A bowl contains white stones numbered 1 to 3 and red stones numbered 1 to 2. Draw a tree diagram to represent total outcomes for selecting two stones.
Topic: Tree Diagrams - Worksheet 5

1. A dice numbered 1 to 5 is rolled and 1 coin tossed. Draw a tree diagram to represent the possible outcomes.

2. Draw a tree diagram to represent total outcomes for flipping 2 dollars.

3. Two colors of bus that are In white and Green color is driven simultaneously. Draw a tree diagram to represent the possible outcomes.

4. A Bag contains Yellow balls numbered 1 to 3, Black balls numbered 1 to 4, and pink balls numbered 1. Draw a tree diagram to represent total outcomes for selecting two balls.

5. Draw a tree diagram to represent total outcomes for flipping a penny.

6. 1 two-sided numbered cube is rolled.
   Draw a tree diagram to represent total outcomes?

7. 2 quartile and one four sides dice numbered from 1 to 4 are tossed. Draw a tree diagram to illustrate total outcomes.

8. If a Shopkeeper has 1 kind of bow in 3 different color red, green and blue. Draw a tree diagram to select.

9. A boy has 2 places to visit.
   Draw a tree diagram to represent the possible outcomes.

10. A jar contains blue pencils numbered 1 to 3 and red pencils numbered 1 to 2. Draw a tree diagram to represent total outcomes for selecting two pencils.
Two-Way Frequency Tables Worksheet

1. Mr. Smith keeps track of his students’ homework completion. He keeps track of how many boys and girls do not complete their homework. He puts students who don’t complete their homework into two categories: first-time offenders and repeat offenders. He uses a table to keep track of the results.

<table>
<thead>
<tr>
<th></th>
<th>First-Time Offenders</th>
<th>Repeat Offenders</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Girls</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   a. In one month 36 girls and 12 boys did not do their homework for the first time. 12 girls and 30 boys did not do their homework again. Put these figures in your table.

   b. How many students did not complete all of their homework assignments this month?

   c. What percentage of the students who did not complete their homework were boys who were First-Time Offenders?

   d. Are boys or girls more likely to not complete their homework? Explain your reasoning.

2. Complete the two-way table for 9th Grader’s school transportation survey:

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walk</td>
<td></td>
<td>46</td>
<td></td>
</tr>
<tr>
<td>Car</td>
<td>28</td>
<td></td>
<td>45</td>
</tr>
<tr>
<td>Bus</td>
<td></td>
<td>12</td>
<td>27</td>
</tr>
<tr>
<td>Bike</td>
<td></td>
<td>17</td>
<td>69</td>
</tr>
<tr>
<td>Total</td>
<td>129</td>
<td>92</td>
<td></td>
</tr>
</tbody>
</table>

   a. What percentage of 9th grade girls walk to school?

   b. What percentage of 9th graders are girls who walk to school?
3. Heather (a hairdresser) is making a record of all the customers she has had in the last month.

   a. Design and label a table that will show the number of male and female customers who are blond or brunette.

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. In one month she has 40 blond females and only 5 blond males. Put these values in the table.

   c. How many blond customers does she have in total that month?

   d. She has 100 total customers and a total of 20 male customers during the same month. Use this information to finish filling in your table.

4. A large group of people was surveyed about their favorite movie genre. The participants had to give their age and choose their favorite genre from Action, Comedy, and Horror.

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Action</th>
<th>Comedy</th>
<th>Horror</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>18-25 years old</td>
<td>238</td>
<td>450</td>
<td>312</td>
<td>1,000</td>
</tr>
<tr>
<td>25-49 years old</td>
<td>350</td>
<td>472</td>
<td>178</td>
<td>1,000</td>
</tr>
<tr>
<td>50+ years old</td>
<td>320</td>
<td>490</td>
<td>190</td>
<td>1,000</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   a. A company that sells a product designed for young adults is looking to advertise before the movies of one of these genres. Which genre should they choose? Explain your reasoning.

   b. If you surveyed 12,000 people total, how many 18-25 year olds would you expect to choose Horror as their favorite genre?

   c. If you surveyed 24,000 people total, how many 25-49 year olds would you expect to choose Comedy as their favorite genre?
Two-Way Frequency Table Worksheet

1. Bob asked 100 adults which one type of music they enjoyed. They could choose Jazz or Rock or Classical or Folk music.

The two-way table shows some information about their answers.

<table>
<thead>
<tr>
<th></th>
<th>Jazz</th>
<th>Rock</th>
<th>Classical</th>
<th>Folk</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>12</td>
<td></td>
<td>19</td>
<td>4</td>
<td>52</td>
</tr>
<tr>
<td>Women</td>
<td></td>
<td>23</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>21</td>
<td></td>
<td>11</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

(a) Complete the two-way table.

(b) How many women did **not** choose Classical music?

2. 60 students were born in England, Scotland or Wales. The two-way table shows some information about these students.

<table>
<thead>
<tr>
<th></th>
<th>England</th>
<th>Scotland</th>
<th>Wales</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td></td>
<td></td>
<td>4</td>
<td>40</td>
</tr>
<tr>
<td>Male</td>
<td></td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>42</td>
<td>12</td>
<td></td>
<td>60</td>
</tr>
</tbody>
</table>

(a) Complete the two-way table.

(b) How many students were **not** born in England.
3. A large group of people was surveyed about their favourite movie genre. The participants had to give their age and choose their favorite genre from Action, Comedy, and Horror.

<table>
<thead>
<tr>
<th></th>
<th>Action</th>
<th>Comedy</th>
<th>Horror</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-25 year olds</td>
<td>238</td>
<td>450</td>
<td>312</td>
<td></td>
</tr>
<tr>
<td>26-49 year olds</td>
<td>350</td>
<td>472</td>
<td></td>
<td>1000</td>
</tr>
<tr>
<td>50+ years old</td>
<td>320</td>
<td></td>
<td>190</td>
<td>1000</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. A company that sells a product designed for teens and young adults is looking to advertise before the movies of one of these genres. Which genre should they choose? Explain.

b. How many total people were surveyed?

c. Which category was least liked by older people?

4. You go to a dance and help clean up afterwards. To help, you collect the soda cans, Coca-Cola and Sprite, and organize them. Some cans were on the table, and some were in the garbage. Seventy-two total cans were found. Forty-two cans were found in the garbage, and 50 total cans were Coca-Cola. 14 Sprite cans were found on the table. Complete the frequency table, and answer the questions.

<table>
<thead>
<tr>
<th></th>
<th>Coca-Cola</th>
<th>Sprite</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Garbage</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. About how many people do you assume were at the party if each person drank 2 cans?

b. If the student plans to recycle all the cans and receives $0.05 for each can, how much money will he make?

5. Make a frequency table based on data you collect. You will need to collect data from females and males in this class. Choose a certain topic you want to research about our class. Possible suggestions are: hair color, eye color, age, iPhone or not, drive to school or not, preferred movie type, preferred music type, preferred soda pop, etc. Conduct your survey, and complete a frequency table.
9.1 TB or Not TB?
A Develop Understanding Task

Tuberculosis (TB) can be tested in a variety of ways, including a skin test. If a person has tuberculosis antibodies, then they are considered to have TB. Below is a tree diagram representing data based on 1,000 people who have been given a skin test for tuberculosis.

1. Use your knowledge to write several probability statements about this test (based on the numbers provided).

2. Look over the statements you wrote. Put an asterisk (*) next to those that are conditional probability statements (statements based on margin "row" or "column" percentages). If there are not any, add some now.

3. Part of understanding the world around us is being able to take information, make sense of it, and then explain it to others. Based on your statements above, what would you say to a friend regarding the validity of their results if they are testing for TB and only get a skin test? Be sure to use data to best inform your friend.
Other questions to consider...

4. In this situation, explain the consequences of errors (having a test with incorrect results).

5. If a health test is not 100% certain, why might it be beneficial to have the results lean more toward a false positive?

6. Is a sample space of 200 enough to indicate whether or not this is true for an entire population?

7. How would you answer the young adult who tested positive and asks, “Do I really have TB?”
Ready, Set, Go!

Ready
Topic: Venn Diagrams, create and read.

For each Venn Diagram provided answer the questions.

1. How many students were surveyed?
2. What were the students asked?
3. How many students are in both choir and band?
4. How many students are not in either choir or band?
5. What is the probability that a randomly selected student would be in band?

This Venn Diagram represents enrollment in some of the elective courses.

6. What does the 95 in the center tell you?
7. What does the 145 tell you?
8. How many total students are represented in the diagram?
9. Which elective class has the least number of students enrolled?
Set

Topic: Interpret a tree diagram, making observations of probability.

Given the tree diagram below answer the questions and determine the probabilities. The diagram represents the number of plate appearances during the first month of a minor league baseball season.

10. How many times did a batter come to the plate during this time period?

11. Based on this data, if you are a left-handed batter what is the probability that you will face a right-handed pitcher?

12. Based on this data, if you are a right-handed batter what is the probability that you will face a left-handed pitcher?

13. What is the probability that a left-handed pitcher will be throwing for any given plate appearance?

14. What is the probability that a left-handed batter would be at the plate for any given plate appearance?

15. What observations do you make about the data? Is there any amount that seems to be overly abundant? What might account for this?

Go

Topic: Basic Probability

Find the probability of achieving success with each of the events below.

16. Rolling an even number on standard six-sided die.

17. Drawing a black card from a standard deck of cards.

18. Flipping a coin and getting Heads three times in a row.

19. Rolling a die and getting a four.

20. Drawing an ace from a deck of cards.

21. Rolling a die twice in a row and getting two threes.

22. From a bag containing 3 blue, 2 red, and 5 white marbles. Pulling out a red marble.
9.2 Chocolate versus Vanilla
_A Solidify Understanding Task_

Danielle loves chocolate ice cream much more than vanilla and was explaining to her best friend Raquel that so does most of the world. Raquel disagreed and thought vanilla is much better. To settle the argument, they created an online survey asking people to choose their favorite ice cream flavor between chocolate and vanilla. After completing the survey, the following results came back:

- There were 8,756 females and 6,010 males who responded.
- Out of all the males, 59.7% chose vanilla over chocolate.
- 4,732 females chose chocolate.

1. Upon first observations, which flavor do you think “won”? ______________. Write a sentence describing what you see at ‘first glance’ that makes you think this.

2. Raquel started to organize the data in the following two-way table. See if you can help complete this (using counts and not percentages):

<table>
<thead>
<tr>
<th></th>
<th>Chocolate</th>
<th>Vanilla</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td></td>
<td>8,756</td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td></td>
<td>6,010</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Organize the same data in a Venn diagram and a tree diagram.
4. Using your organized data representations, write probabilities that help support your claim regarding the preferred flavor of ice cream. For each probability, write a complete statement as well as the corresponding probability notation.

5. Looking over the three representations (tree diagram, two-way table, and Venn diagram), what probabilities seem to be easier to see in each? What probabilities are hidden or hard to see?

<table>
<thead>
<tr>
<th>Highlighted (easier to see)</th>
<th>Hidden</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tree diagram</td>
<td>Tree diagram</td>
</tr>
<tr>
<td>Two-way table</td>
<td>Two-way table</td>
</tr>
<tr>
<td>Venn diagram</td>
<td>Venn diagram</td>
</tr>
</tbody>
</table>

6. Getting back to ice cream. Do you think this is enough information to proclaim the statement that one ice cream is favored over another? Explain.
Ready, Set, Go!

Ready

Topic: Analyzing data in a Venn Diagram

Use the Venn Diagrams below to answer the following questions. (Hint: you may use the same data provided in the two-way table from question 3 on the next page to help make sense of the Venn Diagram)

The following Venn Diagram represents the relationship between favorite sport (Soccer or Baseball) and gender (Female or Male).

1. How many people said soccer is their favorite sport?
2. How many females are in the data?
3. How many males chose baseball?
4. What is the probability that a person would say soccer is their favorite sport? $P(\text{soccer}) =

5. What is the probability that a female would say soccer is their favorite sport? ("Out of all females, ___% say soccer is their favorite sport") $P(\text{soccer} \mid \text{female}) =

The following Venn Diagram represents the relationship between favorite subject (Math or Science) and grade level (Ninth or Tenth). Using this data, answer the following questions.

6. How many people said math is their favorite subject?
7. How many tenth graders are in the data?
8. How many ninth graders chose science?
9. What is the probability that a person would say science is their favorite subject? $P(s) = 30$

10. What is the probability that a tenth grader would say science is their favorite subject? ("If you are a tenth grader, then the probability of science being your favorite subject is ___ %") $P(\text{science} \mid \text{tenth}) =
Set

Topic: Writing conditional statements from a two-way table.

11. Complete the table and write three conditional statements.

<table>
<thead>
<tr>
<th></th>
<th>Soccer</th>
<th>Baseball</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td></td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>50</td>
<td></td>
<td>76</td>
</tr>
<tr>
<td>Total</td>
<td>85</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

12. Complete the table about preferred genre of reading and write three conditional statements.

<table>
<thead>
<tr>
<th></th>
<th>Fiction</th>
<th>Non-Fiction</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td></td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>50</td>
<td></td>
<td>60</td>
</tr>
<tr>
<td>Total</td>
<td>85</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

13. Complete the table about favorite color of M&M's and write three conditional statements.

<table>
<thead>
<tr>
<th></th>
<th>Blue</th>
<th>Green</th>
<th>Red</th>
<th>Other</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>15</td>
<td>20</td>
<td>15</td>
<td></td>
<td>60</td>
</tr>
<tr>
<td>Female</td>
<td>30</td>
<td>20</td>
<td></td>
<td>10</td>
<td>130</td>
</tr>
</tbody>
</table>

14. Use the information provided to make a tree diagram, a two-way table and a Venn Diagram.
   - Data was collected at the movie theater last fall. Not about movies but clothes.
   - 6,525 people were observed.
   - 3,123 had on shorts and the rest had on pants
   - 45% of those wearing shorts were denim.
   - Of those wearing pants 88% were denim.
Go

Topic: Fractions, Percent and there operations

Find the desired values.

15. What is half of one-third?  16. What is one-third of two-fifths?

17. What is one-fourth of four-sevenths?  18. What percent is $\frac{5}{8}$?

19. What is 35% of 50?  20. Seventy is 60% of what number?

21. Write $\frac{7}{12}$ as a percent.  22. Write $\frac{1}{6}$ as a percent.

23. What is 52% of 1,200?  24. What percent is 32 of 160?

25. Sixty is what percent of 250?  26. What percent of 350 is 50?
9.3 Fried Freddy’s  
A Solidify Understanding Task

Danielle was surprised by the results of the survey to determine the ‘favorite ice cream’ between chocolate and vanilla (See task 9.2 Chocolate vs. Vanilla). The reason, she explains, is that she had asked several of her friends and the results were as follows:

<table>
<thead>
<tr>
<th></th>
<th>Chocolate</th>
<th>Vanilla</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>23</td>
<td>10</td>
<td>33</td>
</tr>
<tr>
<td>Male</td>
<td>6</td>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>Total</td>
<td>29</td>
<td>18</td>
<td>47</td>
</tr>
</tbody>
</table>

1. In this situation, chocolate is most preferred. How would you explain to her that this data may be less ‘valid’ compared to the data from the online survey?

Using a sufficiently large number of trials helps us estimate the probability of an event happening. If the sample is large enough, we can say that we have an estimated probability outcome for the probability of an event happening. If the sample is not randomly selected (only asking your friends) or not large enough (collecting four data points is not enough information to estimate long run probabilities), then one should not estimate large scale probabilities. Sometimes, our sample increases in size over time. Below is an example of data that is collected over time, so the estimated probability outcome becomes more precise as the sample increases over time.

Freddy loves fried food. His passion for the perfect fried food recipes led to him opening the restaurant, “Fried Freddies.” His two main dishes are focused around fish or chicken. Knowing he also had to open up his menu to people who prefer to have their food grilled instead of fried, he created the following menu board:
After being open for six months, Freddy realized he was having more food waste than he should because he was not predicting how much of each he should prepare in advance. His business friend, Tyrell, said he could help.

2. What information do you think Tyrell would need?

Luckily, Freddy uses a computer to take orders each day so Tyrell had lots of data to pull from. After determining the average number of customers Freddy serves each day, Tyrell created the following Venn diagram to show Freddy the food preference of his customers:

To make sense of the diagram, Freddy computed the following probability statements:

3. What is the probability that a randomly selected customer would order fish?
\[ P(\text{fish}) = \]
Shade the part of the diagram that models this solution.

4. What is the probability that a randomly selected customer would order fried fish?
\[ P(\text{fried } \cap \text{ fish}) = P(\text{fried and fish}) = \]
Shade the part of the diagram that models this solution.

5. What is the probability that a person prefers fried chicken?
\[ P(\text{fried } \cap \text{ chicken}) = P(\text{fried and chicken}) = \]
Shade the part of the diagram that models this solution.
6. What is the estimated probability that a randomly selected customer would want their fish grilled?
   \[ P(\text{grilled and fish}) = P(\underline{\quad}) = \]
   Shade the part of the diagram that models this solution.

7. If Freddy serves 100 meals at lunch on a particular day, how many orders of fish should he prepare with his famous fried recipe?

8. What is the probability that a randomly selected person would choose fish or fried?
   \[ P(\text{fried U fish}) = P(\text{fried or fish}) = \]
   Shade the part of the diagram that models this solution.

9. What is the probability that a randomly selected person would NOT choose fish or fried?
   Shade the part of the diagram that models this solution.
Ready, Set, Go!

Ready

Topic: Independent and Dependent events

In some of the situations described below the first event affects the subsequent event (dependent events). In others each of the events is completely independent of the others (independent events). Determine which situations are dependent and which are independent.

1. A coin is flipped twice. The first event is the first flip and the second event is the next flip.

2. A bag of marbles contains 3 blue marbles, 6 red marbles and 2 yellow marbles. Two of the marbles are drawn out of the bag. The first even is the first marble taken out the second event is the second marble taken out.

3. An attempt to find the probability of there being a right-handed or a left-handed batter a the plate in a baseball game. The first event is the 1st batter to come to the plate. The second event is the second player to come up to the plate.

4. A standard die is rolled twice. The first event is the first roll and the second event is the second roll.

5. Two cards are drawn from a standard deck of cards. The first event is the first card that is drawn the second event is the second card that is drawn.

Set

Topic: Addition Rule, interpreting a Venn Diagram

6. Sally was assigned to create a Venn diagram to represent \( P(A \text{ or } B) \). Sally first writes \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \), what does this mean? Explain each part.

7. Sally then creates the following diagram. Sally’s Venn diagram is incorrect. Why?
The Venn diagram to the right shows the data collected at a sandwich shop for the last six months with respect to the type of bread people ordered (sourdough or wheat) and whether or not they got cheese on their sandwich. Use this data to create a two-way frequency table and answer the questions.

8. Two-way frequency table

<table>
<thead>
<tr>
<th></th>
<th>with cheese</th>
<th>without cheese</th>
</tr>
</thead>
<tbody>
<tr>
<td>sour</td>
<td>55%</td>
<td>10%</td>
</tr>
<tr>
<td>wheat</td>
<td>20%</td>
<td>5%</td>
</tr>
</tbody>
</table>

9. What is the probability that a randomly selected customer would order sourdough bread?
   \[ P(\text{sourdough bread}) = \]

10. What is the probability that a randomly selected customer would order sourdough bread without cheese?
    \[ P(\text{sourdough } \cap \text{ no cheese}) = P(\text{sourdough and no cheese}) = \]

11. What is the probability that a person prefers wheat bread without cheese?
    \[ P(\text{wheat } \cap \text{ no cheese}) = P(\text{wheat and no cheese}) = \]

12. What is the estimated probability that a randomly selected customer would want their sandwich with cheese?
    \[ P(\text{sourdough cheese and wheat cheese}) = P(\text{______________}) = \]

13. If they serve 100 sandwiches at lunch on a particular day, how many orders with sourdough should be prepared without cheese?

14. What is the probability that a randomly selected person would choose sourdough or without cheese?
    \[ P(\text{sourdough } \cup \text{ no cheese}) = P(\text{sourdough or no cheese}) = \]

15. What is the probability that a randomly selected person would NOT choose sourdough or no cheese?
Go

Topic: Equivalent ratios and proportions

Use the given ratio to set up a proportion and find the desired value.

16. If 3 out of 5 students eat school lunch then how many students would be expected to eat school lunch at a school with 750 students?

17. In a well developed and carried out survey it was found that 4 out of 10 students have a pair of sunglasses. How many students would you expect to have a pair of sunglasses out of a group of 45 students?

18. Data collected at a local mall indicated that 7 out of 20 men observed were wearing a hat. How many would you expect to have been wearing hats if 7500 men were to be at the mall on a similar day?
9.4 Visualizing with Venn
A Solidify Understanding Task

One of the attributes of Venn diagram's is that it can be easy to see the relationships within the data. In this task, we will create multiple Venn diagrams using data and determine the events that create diagrams to either have an intersection or for them to be mutually exclusive.

1. The following data represents the number of men and women passengers aboard the titanic and whether or not they survived.

<table>
<thead>
<tr>
<th></th>
<th>Survived</th>
<th>Did not survive</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>146</td>
<td>659</td>
<td>805</td>
</tr>
<tr>
<td>Women</td>
<td>296</td>
<td>106</td>
<td>402</td>
</tr>
<tr>
<td>Total</td>
<td>442</td>
<td>765</td>
<td>1207</td>
</tr>
</tbody>
</table>

2. Create three Venn diagrams with this data.
   a. Men vs Women
   b. Women vs Survived
   c. You choose the conditions

3. Create two probability statements using each of your Venn diagrams from question 2.

4. Create and label three different Venn diagrams using this data. Create at least one that is mutually exclusive and at least one that has an intersection.
   Sample size: 100
   \[
   P(\text{girl}) = \frac{42}{100}
   \]
   \[
   P(\text{girl or art}) = (\frac{42}{100} + \frac{30}{100}) - \frac{12}{100}
   \]
   \[
   P(\text{art}) = \frac{30}{100}
   \]
   \[
   P(\text{not art}) =
   \]
   \[
   P(\text{boy}) =
   \]

5. Describe the conditions that create mutually exclusive Venn diagrams and those that create intersections.

6. What conjecture can you make regarding the best way to create a Venn diagram from data to highlight probabilities?
Ready, Set, Go!

Ready
Topic: Products of probabilities, multiplying and dividing fractions

Find the products or quotients below.

1. \( \frac{1}{2} \cdot \frac{2}{3} \)
2. \( \frac{3}{5} \cdot \frac{1}{3} \)
3. \( \frac{7}{10} \div \frac{2}{5} \)
4. \( \frac{8}{7} \cdot \frac{3}{4} \)
5. \( \frac{1}{2} \div \frac{3}{1} \)
6. \( \frac{2}{5} \div \frac{2}{3} \)

7. \( P(A) = \frac{3}{4} \)
   \( P(B) = \frac{1}{2} \)
   \( P(A) \times P(B) = \)

8. \( P(A) = \frac{1}{6} \)
   \( P(B) = \frac{1}{3} \)
   \( P(A) \times P(B) = \)

Set
For each situation, one of the representations (two-way table, Venn diagram, tree diagram, context or probability notation) is provided. Use the provided information to complete the remaining representations.

9. Are you Blue?

<table>
<thead>
<tr>
<th>Notation</th>
<th>2-way Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>Key:</td>
<td></td>
</tr>
<tr>
<td>Male = M</td>
<td>Female = F</td>
</tr>
<tr>
<td>Blue = B</td>
<td>Not Blue = N</td>
</tr>
<tr>
<td>Sample size = 200</td>
<td></td>
</tr>
<tr>
<td>( P(B) = \frac{84}{200} )</td>
<td>( P(M) = \frac{64}{200} )</td>
</tr>
<tr>
<td>( P(F</td>
<td>B) = \frac{48}{84} )</td>
</tr>
<tr>
<td>( P(M \cap B) = )</td>
<td>( P(M \cup B) = )</td>
</tr>
</tbody>
</table>

Blue | Not Blue | Total |
--- | --- | --- |
Male | | |
Female | | |
Total | | |

(Continued on the next page)
(Continued from the last page)

<table>
<thead>
<tr>
<th>Venn Diagram</th>
<th>Tree Diagram</th>
</tr>
</thead>
</table>

Write three observations you can make about this data.

10. Right and left handedness of a group.

<table>
<thead>
<tr>
<th>Notation</th>
<th>2-way Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>Key: Male = M</td>
<td>Female = F</td>
</tr>
<tr>
<td>Lefty = L</td>
<td>Righty = R</td>
</tr>
</tbody>
</table>

Sample size = 100 people

\[
P(L) = \quad P(M) = \]

\[
P(F) = \quad P(L|F) = \]

\[
P(L|M) = \]

<table>
<thead>
<tr>
<th>Venn Diagram</th>
<th>Tree Diagram</th>
</tr>
</thead>
</table>

Write three conditional statements regarding this data.
11. The most important meal of the day.

<table>
<thead>
<tr>
<th>Notation</th>
<th>2-way Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>Key: Male = M Female = F Eats Breakfast = E Doesn’t Eat Breakfast = D</td>
<td></td>
</tr>
<tr>
<td>Sample size =</td>
<td>Eats</td>
</tr>
<tr>
<td>P(E) =</td>
<td>Male</td>
</tr>
<tr>
<td>P(E</td>
<td>M) =</td>
</tr>
<tr>
<td>P(E\M)=</td>
<td>Total</td>
</tr>
<tr>
<td>P(E\F) =</td>
<td></td>
</tr>
</tbody>
</table>

Venn Diagram

Tree Diagram

Does this data surprise you? Why or why not.

---

**Go**

Topic: Writing conditional statements from a two-way table.

12. Complete the table and write three conditional statements.

<table>
<thead>
<tr>
<th></th>
<th>Biking</th>
<th>Swimming</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td></td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>35</td>
<td></td>
<td>76</td>
</tr>
<tr>
<td>Total</td>
<td>85</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

13. Complete the table about preferred genre of reading and write three conditional statements.

<table>
<thead>
<tr>
<th></th>
<th>Ice Cream</th>
<th>Cake</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td></td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>10</td>
<td></td>
<td>60</td>
</tr>
<tr>
<td>Total</td>
<td>85</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

14. Complete the table about eye color and write three conditional statements.

<table>
<thead>
<tr>
<th></th>
<th>Blue</th>
<th>Green</th>
<th>Brown</th>
<th>Other</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>55</td>
<td>20</td>
<td>15</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>Female</td>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>75</td>
<td>20</td>
<td></td>
<td></td>
<td>230</td>
</tr>
</tbody>
</table>
9.5 Freddy Revisited
A Solidify Understanding Task

Once Tyrell helped Freddy out in determining the amount and type of food Freddy should prepare each day for his restaurant, Freddy's food waste decreased dramatically. Still, Freddy noticed that during the week, he seemed to still have more food prepared than he needed, and sometimes on the weekend he would run out of something he needed. Tyrell said another level of determining waste could be if Freddy averaged the number of orders he received of fish and chicken on a weekday and compared it to the average number of orders he received of each on the weekend. Freddy thought this was a good idea so started collecting data. After two months, he had enough information to create a two way table representing the average number of orders he received on the weekdays and on the weekends for fish and chicken. The data is below:

<table>
<thead>
<tr>
<th></th>
<th>Chicken</th>
<th>Fish</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekday</td>
<td>65</td>
<td>79</td>
<td>144</td>
</tr>
<tr>
<td>Weekend</td>
<td>88</td>
<td>107</td>
<td>195</td>
</tr>
<tr>
<td>Total</td>
<td>153</td>
<td>186</td>
<td>339</td>
</tr>
</tbody>
</table>

1. What observations can you make? Explain to Freddy what this means (When does Freddy seem to have the greatest business? Should he expect a greater percentage of customers to order fish during the week or on the weekend? What else?)

2. Does the number of orders of chicken compared to fish depend on whether it is a weekday or a weekend? What values from the table tell you this?
Ready, Set, Go!

Ready
Topic: Quadratic Function Review

Find the x-intercepts, y-intercept, line of symmetry and vertex for the quadratic functions.
1. \( f(x) = x^2 + 8x - 9 \)  
2. \( g(x) = x^2 - 3x - 5 \)  
3. \( h(x) = 2x^2 + 5x - 3 \)

4. \( k(x) = x^2 + 6x - 9 \)  
5. \( p(x) = (x + 5)^2 - 2 \)  
6. \( q(x) = (x + 7)(x - 5) \)

Set
Topic: Independence

Determining the independence of events can sometimes be done by becoming familiar with the context in which the events occur and the nature of the events. There are also some ways of determining independence of events based on equivalent probabilities.

- Two events, \( A \) and \( B \), are independent if \( P(A \text{ and } B) = P(A) \cdot P(B) \)
- Additionally, two events, \( A \) and \( B \), are independent if \( P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = P(A) \)

Use these two ways of determining independent events to determine independence in the problems below and answer the questions.

7. \( P(A \text{ and } B) = \frac{2}{5} \)  
   \( P(A) = \frac{1}{2} \)  
   \( P(B) = \frac{3}{10} \)

8. \( P(A) = \frac{1}{5} \)  
   \( P(A \text{ and } B) = \frac{1}{6} \)  
   \( P(B) = \frac{1}{3} \)
9. \[ P(A) = \frac{1}{2} \]
\[ P(A \text{ and } B) = \frac{1}{5} \]
\[ P(B) = \frac{2}{5} \]

10. \[ P(A \text{ and } B) = \frac{2}{5} \]
\[ P(A) = \frac{1}{4} \]
\[ P(B) = \frac{4}{5} \]

**Go**

Topic: Find probabilities from a two-way table.

The following data represents the number of men and women passengers aboard the titanic and whether or not they survived.

<table>
<thead>
<tr>
<th></th>
<th>Survived</th>
<th>Did not survive</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>146</td>
<td>659</td>
<td>805</td>
</tr>
<tr>
<td>Women</td>
<td>296</td>
<td>106</td>
<td>402</td>
</tr>
<tr>
<td>Total</td>
<td>442</td>
<td>765</td>
<td>1207</td>
</tr>
</tbody>
</table>

11. \( P(w) = \)

12. \( P(s) = \)

13. \( P(s|w) = \)

14. \( P(w \text{ or } s) = \)

15. \( P(w \text{ or } m) = \)

16. \( P(ns|w) = \)

17. \( P(m \cap ns) = \)
9.6 Striving for Independence

A Practice Understanding Task

Use your knowledge of conditional probability (the probability of $A$ given $B$ as $P(A \text{ and } B)/P(B)$) as well as the definition of independence (two events ($A$ and $B$)) are said to be independent if $P(A|B) = P(A)$ and $P(B|A) = P(B)$) to answer the following questions. Keep track of how you are determining independence for each type of representation.

1. Out of the 2000 students who attend a certain high school, 1400 students own cell phones, 1000 own a tablet, and 800 have both. Suppose a student is randomly selected. Create a Venn diagram model and use notation to answer the following questions.

   a) What is the probability that a randomly selected student owns a cell phone?

   b) What is the probability that a randomly selected student owns both a cell phone and a tablet?

   c) If a randomly selected student owns a cell phone (was one of the 1400 with a phone), what is the probability that this student also owns a tablet?

   d) How are questions c and d different?

   e) Are the outcomes own a cell phone and own a tablet independent? Explain.

   f) If question e is not independent, what number of students would own a tablet to create independence?

2. Below is a partially completed tree diagram from the task Chocolate vs Vanilla.

   a) Circle the parts of the diagram you would use to determine if choosing chocolate is independent of being a male or female.

   b) Complete the diagram so that choosing chocolate is independent of being male or female.
3. Use the titanic data below to answer the following questions.

<table>
<thead>
<tr>
<th></th>
<th>Survived</th>
<th>Did not survive</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>146</td>
<td>659</td>
<td>805</td>
</tr>
<tr>
<td>Women</td>
<td>296</td>
<td>106</td>
<td>402</td>
</tr>
<tr>
<td>Total</td>
<td>442</td>
<td>765</td>
<td>1207</td>
</tr>
</tbody>
</table>

a. Determine if survival was independent of gender. Explain.

b. If gender would not have mattered, what would have been the number of males that would have survived, given the data for the number of females who survived and the total number of passengers on the ship.

4. Determine whether the second scenario would be dependent or independent of the first scenario. Explain.

a) Rolling a six-sided die, then drawing a card from a deck of 52 cards.
b) Drawing a card from a deck of 52 cards, then drawing another card from the same deck.
c) Rolling a six-sided die, then rolling it again.
d) Pulling a marble out of a bag, replacing it, then pulling a marble out of the same bag.
e) Having 20 treats in five different flavors for a soccer team, with each player taking a treat.

5. The definition of independence is that two events (A and B) are said to be independent if

\[ P(A|B) = P(A) \text{ and } P(B|A) = P(B) \]

Explain what this looks like in a Venn diagram, a tree diagram, and a two-way table.
Ready, Set, Go!

Ready
Topic: End of year Review

Solve each of the quadratics below using an appropriate method.

1. \( m^2 + 15m + 56 = 0 \)
2. \( 5x^2 - 3x + 7 = 0 \)
3. \( x^2 - 10x + 21 = 0 \)
4. \( 6x^2 + 7x - 5 = 0 \)

Set
Topic: Representing Independent Events in Venn Diagrams

In each of the Venn Diagrams the number of outcomes for each event are given, use the provided information to determine the conditional probabilities or independence. The numbers in the Venn Diagram indicate the number of outcomes in that part of the sample space.

5. a. How many total outcomes are possible?
   b. \( P(A) = \)
   c. \( P(B) = \)
   d. \( P(A \cap B) = \)
   e. \( P(A|B) = \)
   f. Are events A and B independent events? Why or why not?
6. a. How many total outcomes are possible?
b. \( P(E) = \)
c. \( P(F) = \)
d. \( P(E \cap F) = \)
e. \( P(E|F) = \)

f. Are events E and F independent events? Why or why not?

7. a. How many total outcomes are possible?
b. \( P(X) = \)
c. \( P(Y) = \)
d. \( P(X \cap Y) = \)
e. \( P(X|Y) = \)

f. Are events X and Y independent events? Why or why not?

8. a. How many total outcomes are possible?
b. \( P(K) = \)
c. \( P(L) = \)
d. \( P(K \cap L) = \)
e. \( P(K|L) = \)

f. Are events K and L independent events? Why or why not?
Go

Topic: Conditional Probability and Independence

Data gathered on the shopping patterns during the months of April and May of high school students from Peanut Village revealed the following. 38% of students purchased a new pair of shorts (call this event H), 15% of students purchased a new pair of sunglasses (call this event G) and 6% of students purchased both a pair of short and a pair of sunglasses.

9. Find the probability that a student purchased a pair of sunglasses given that you know they purchased a pair of shorts. \( P(G|H) = \)

10. Find the probability that a student purchased a pair of shorts or purchased a new pair of sunglasses. \( P(H \cup G) = \)

11. Given the condition that you know a student has purchased at least one of the items. What is the probability that they purchased only one of the items?

12. Are the two events H and G independent of one another? Why or Why not?

Given the data collected from 200 individuals concerning whether or not to extend the length of the school year in the table below answer the questions.

<table>
<thead>
<tr>
<th></th>
<th>For</th>
<th>Against</th>
<th>No Opinion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Youth (5 to 19)</td>
<td>7</td>
<td>35</td>
<td>12</td>
</tr>
<tr>
<td>Adults (20 to 55)</td>
<td>30</td>
<td>27</td>
<td>20</td>
</tr>
<tr>
<td>Seniors (55 +)</td>
<td>25</td>
<td>16</td>
<td>28</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>200</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

13. Given that condition that a person is an adult what is the probability that they are in favor of extending the school year? \( P(\text{For|Adult}) = \)

14. Given the condition that a person is against extending the school year what is the probability they are a Senior? \( P(\text{Senior|Against}) = \)

15. What is the probability that a person has no opinion given that they are a youth? \( P(\text{no opinion|youth}) = \)

Need Assistance? Check out these additional resources:
http://www.youtube.com/watch?v=r6g0hMldw48M

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